Solved by A. Alt; G. Apostolopoulos; Š. Arslanagić; M. Bataille; C. Curtis; O. Geupel; O. Kouba; S. Malikić; Missouri State University Problem Solving Group; D. Smith; I. Uchiha; and the proposer. In addition, three submissions were incorrect and one was incomplete. We present a composite of solutions by Salem Malikić and Itachi Uchiha.

Let (x, y) be the coordinates of a variable point P of the ellipse in the first quadrant; that is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x, y > 0$. The line OP has slope $\frac{y}{x}$ while the tangent to the ellipse at P has slope -$\frac{b^2x}{a^2y}$, so the acute angle between these lines satisfies (with the help of the AM-GM inequality)

$$\tan\theta = \frac{\frac{y}{x} - \left(-\frac{b^2x}{a^2y}\right)}{1 - \frac{b^2}{a^2}} = \frac{1}{a^2 - b^2}\left(\frac{a^2y}{x} + \frac{b^2x}{y}\right) \geq \frac{2ab}{a^2 - b^2}.$$

Equality holds if and only if $\frac{a^2y}{x} = \frac{b^2x}{y}$; that is, if and only if $\frac{x^2}{a^2} = \frac{y^2}{b^2}$. But in the equation of the ellipse these equal fractions sum to 1, so they must each equal $\frac{1}{2}$. Since the tangent function is strictly increasing, $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ is the point where the acute angle θ achieves its minimum.

3786. [2012 : 379, 380] *Proposed by Mehmet Şahin.*

Let ABC be a triangle with medians m_a, m_b and m_c, circumradius R and inradius r. Let P be the point of intersection of the bisector of $\angle A$ and the median from B, Q be the point of intersection of the bisector of $\angle B$ and the median from C, and R be the point of intersection of the bisector of $\angle C$ and the median from A. If $\angle APB = \alpha$, $\angle BQC = \beta$ and $\angle CRA = \gamma$, prove that

$$\frac{m_a m_b m_c \sin\alpha \sin\beta \sin\gamma}{(a + 2b)(b + 2c)(c + 2a)} = \frac{r}{32R}\ .$$

Solved by A. Alt; AN-anduud Problem Solving Group; G. Apostolopoulos; M. Bataille; C. Curtis; J. G. Heuver; D. Koukakis; S. Malikić; C. R. Pranesachar; D. Văcaru; P. Y. Woo; T. Zvonaru; and the proposer. We present a composite solution.

Let M be the midpoint of AC. Because AP bisects $\angle A$ in ΔABM, we have $\frac{BP}{PM} = \frac{AB}{AM} = \frac{c}{b/2}$, whence

$$\frac{BP}{2c} = \frac{PM}{b} = \frac{BP + PM}{2c + b} = \frac{m_b}{b + 2c}.$$

From the sine law applied to ΔABP, $\frac{BP}{c} = \frac{\sin A/2}{\sin\alpha}$, which yields

$$\frac{m_b \sin\alpha}{b + 2c} = \frac{1}{2}\sin\frac{A}{2}.$$

Similarly,

$$\frac{m_c \sin\beta}{c+2a} = \frac{1}{2}\sin\frac{B}{2} \quad \text{and} \quad \frac{m_a \sin\gamma}{a+2b} = \frac{1}{2}\sin\frac{C}{2}.$$

By multiplication it follows that

$$\frac{m_a m_b m_c \sin\alpha \sin\beta \sin\gamma}{(a+2b)(b+2c)(c+2a)} = \frac{1}{8}\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.$$

The desired result follows immediately from the identity

$$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{r}{4R},$$

which holds for all triangles ABC.

3787. [2012 : 379, 381] *Proposed by Michel Bataille.*

Let S be a finite set with cardinality $|S| = n \geq 1$ and let k be a positive integer. Calculate

$$\sum_{(A)} |A(1) \cap A(2) \cap \cdots \cap A(k)| \text{ and } \sum_{(A)} |A(1) \cup A(2) \cup \cdots \cup A(k)|,$$

where the summation $\sum_{(A)}$ is over all mappings A from $\{1, 2, \ldots, k\}$ to the power set $\mathcal{P}(S)$.

Solved by AN-anduud Problem Solving Group ; O. Geupel ; O. Kouba ; Missouri State University Problem Solving Group ; C. R. Pranesachar ; and the proposer. Skidmore College Problem Group provided correct solution but without proof. We present the solution by Oliver Geupel.

Let us introduce convenient notation :

$$f(n,k) = \sum_{(A)} |A(1) \cap A(2) \cap \cdots \cap A(k)|,$$
$$g(n,k) = \sum_{(A)} |A(1) \cup A(2) \cup \cdots \cup A(k)|.$$

We prove that

$$f(n,k) = n \cdot 2^{k(n-1)}, \tag{1}$$
$$g(n,k) = n \cdot 2^{k(n-1)}(2^k - 1). \tag{2}$$

Without loss of generality, we may assume that $S = \{1, 2, \ldots, n\}$. We let $\mathcal{F}$ denote the set of all mappings $A : \{1, 2, \ldots, k\} \to \mathcal{P}(S)$. Then for any $A \in \mathcal{F}$, we define a $k \times n$ matrix $M(A) = (a_{ij})$ such that

$$a_{ij} = \begin{cases} 0 & \text{if } j \notin A(i), \\ 1 & \text{if } j \in A(i). \end{cases}$$